Can you count to 31 on one hand?

How high can you count using only one hand (five fingers)?

Hold out your hand with your pinky on the right side, thumb on the left.

- Pinky finger = 1
- Ring finger = 2
- Middle finger = 4
- Pointer finger = 8
- Thumb = 16

For example, if you hold up only your

- Pinky + ring = $1 + 2 = 3$
- Pinky + thumb = $1 + 16 = 17$
- Ring + middle + pointer = $2 + 4 + 8 = 14$

To write in binary: 0 means a finger is down, 1 means up. For example, 01100 is pointer and middle fingers up; 11001 is thumb, pointer, and pinky up.

So the first few counting numbers are

0: 00000 (all fingers down)

1: 00001 (only pinky up)

2: 00010 (only ring finger up)

3: 00011

4: 00100

5: 00101

Can you keep going? All the way to 31?

Can you count from 1 to 31 on your fingers in one minute?

This counting system is called **binary**, or base two. Usually we use base ten, where we have a ones column, tens column, hundreds, thousands, etc. For base n, you have a ones column, n column, n^2 column, n^3, etc. (When $n = 2$, $n^2 = 4$ and $n^3 = 8$; when $n = 10$, $n^2 = 100$ and $n^3 = 1{,}000$.)

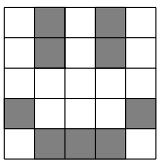
Bonus challenge: how high can you count using both hands (10 fingers)?

Encode your own pictures

Introduction: computers store not only numbers but words, pictures, music, and videos, all using binary. Here you'll learn how to encode and decode small pictures, so you can send secret picture messages.

Instructions for encoding:

I1. Draw your picture in a grid below (or make your own: 5 columns, 5 rows); each little "pixel" square must be totally white or totally filled (gray/black).
I2. In another grid, write 1 for each pixel that's dark, and write 0 for each white pixel.
I3. For the first row: translate the number from binary (base 2) to decimal (base 10, our usual numbers, like 17 means "seventeen"). Do the same for the next four rows, so you have five numbers total.
I4. These five numbers represent your picture; give them to a friend (or to Mr. Allee or Dr. Dave) to see if they can decode your secret picture!

Example encoding:  $\Rightarrow$

16	8	4	2	1
0	1	0	1	0
0	1	0	1	0
0	0	0	0	0
1	0	0	0	1
0	1	1	1	0

$\Rightarrow$

$8+2=10$
$8+2=10$
0
$16+1=17$
$8+4+2=14$

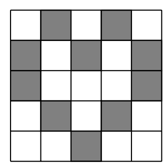
$\Rightarrow$ encoded image is $(10, 10, 0, 17, 14)$

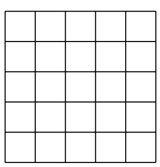
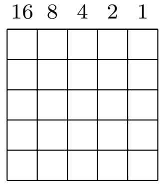
Example decoding: $(10, 21, 17, 10, 4) \Rightarrow$

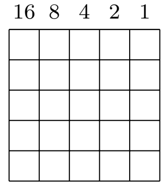
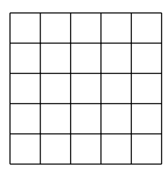
$10=8+2$
$21=16+4+1$
$17=16+1$
$10=8+2$
4

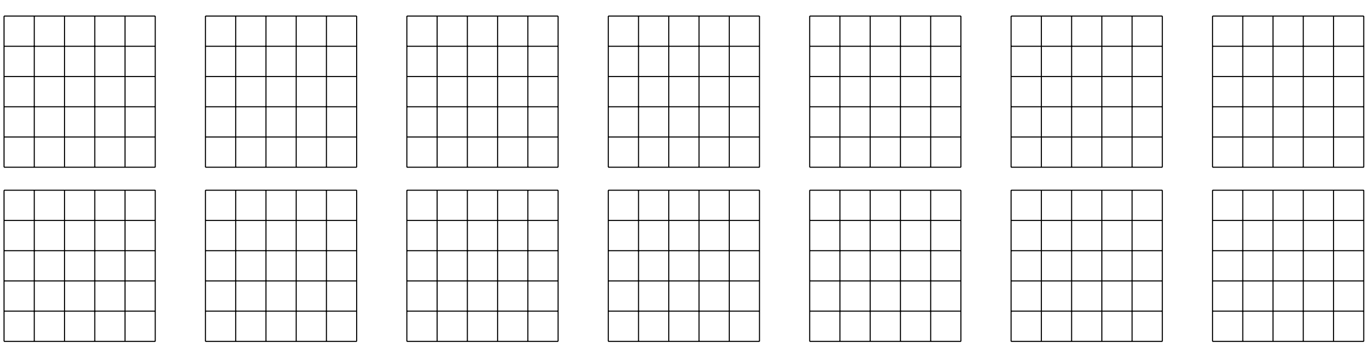
$\Rightarrow$

16	8	4	2	1
0	1	0	1	0
1	0	1	0	1
1	0	0	0	1
0	1	0	1	0
0	0	1	0	0

$\Rightarrow$ 

Encode your own picture!  $\Rightarrow$ 16 8 4 2 1  $\Rightarrow$ (___ , ___ , ___ , ___ , ___)

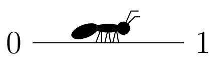
Decode someone else's! (___ , ___ , ___ , ___ , ___) $\Rightarrow$ 16 8 4 2 1  $\Rightarrow$ 



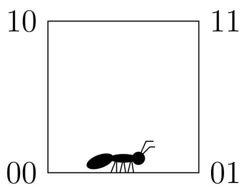
Binary corners

1. Imagine this dot . is the entire world (universe?) of an ant. Because the ant has only one place to be, she* does not need to label different places with addresses. (*Almost all ants you ever see are female.)

2. (One-digit world) Imagine instead two dots connected by a line — is where the ant lives. Now she can be on the left end or the right end (and walk along the line to move between the dots), so she needs addresses! She labels the left end 0 and the right end 1:



4. (Two-digit world) Imagine an even bigger upgrade, doubling the ant's world again: two of the two-dot lines, where both left ends are connected, and both right ends are connected:



The second digit shows left (0) or right (1).
The first digit shows bottom (0) or top (1).
Examples: 01 means bottom-right; 11 means top-right.

8. (Three-digit world) What comes next? Take one square and add a 0 in front of each label (000, 001, 010, 011). Take another square but add a 1 in front of each label (100, 101, 110, 111). Connect the corresponding corners: connect 000 to 100, 001 to 101, 010 to 110, and 011 to 111. Can you draw it? If you make it 3-dimensional, is there a name for it?

16. (Four-digit world) What comes next??

Questions:

- V1. Each label is on a "vertex" (like a corner). In the one-digit world (line), there are two vertices: 0 and 1. How many vertices are there in the two-digit world (square)?
- V2. How many vertices are there in the three-digit world?
- V3. Do you see a pattern in the number of vertices? Describe it.
- V4. How many vertices are there if you have n digits?

- D1. In the one-digit world (line), the ant only needs to travel along one "edge" to get from vertex 0 to vertex 1. In the two-digit world (square), how many edges does the ant need to travel to get from vertex 00 to 11?
- D2. In the three-digit world: how many edges does the ant need to travel to get from 000 to 111?
- D3. In the n-digit world: how many edges does the ant need to travel to get from 00000… to 11111…?

- P1. Imagine the ants are not social insects but instead anti-social. They do not want to live only one edge away from another ant. How many ant houses can you fit in the two-digit world?
- P2. In the three-digit world, how many ant houses can you fit, without having any that are only one edge apart?
- P3. In the four-digit world, how many can you fit?