Introduction

Fractals are mathematical shapes that have an interesting structure no matter how far you "zoom in," and the structural patterns look similar no matter how far you zoom in. With math, we can think about infinitely complex fractals. In nature, there are many patterns that are fractal-like, from clouds to coastlines to river basins to romanesco (a type of cauliflower). Both mathematical and natural fractal structures can be beautiful.



The following pages introduce you to some classic fractal patterns. Work on whichever parts most interest you: there is drawing, geometry, and number patterns. As always, if you need help or want a hint, just raise your hand!

The "rulers" at the bottom of this page may be helpful for the drawing parts, when you need to divide a line segment into two (top ruler) or three (bottom ruler) equal segments. It will probably help if you **fold the bottom of this page under** (fold along either of the two horizontal lines below, depending which ruler you want to use), so the ruler is at the very edge. (If you're not sure what I mean, just raise your hand and I'll come show you.)

Suggestion: instead of trying to draw a triangle/square, first mark the corners, then connect the corners with straight lines. As you'll see in the templates, the corners are found by dividing each line segment into either two or three equal pieces (hence the rulers below).

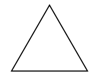
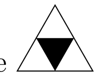
Enjoy! :)

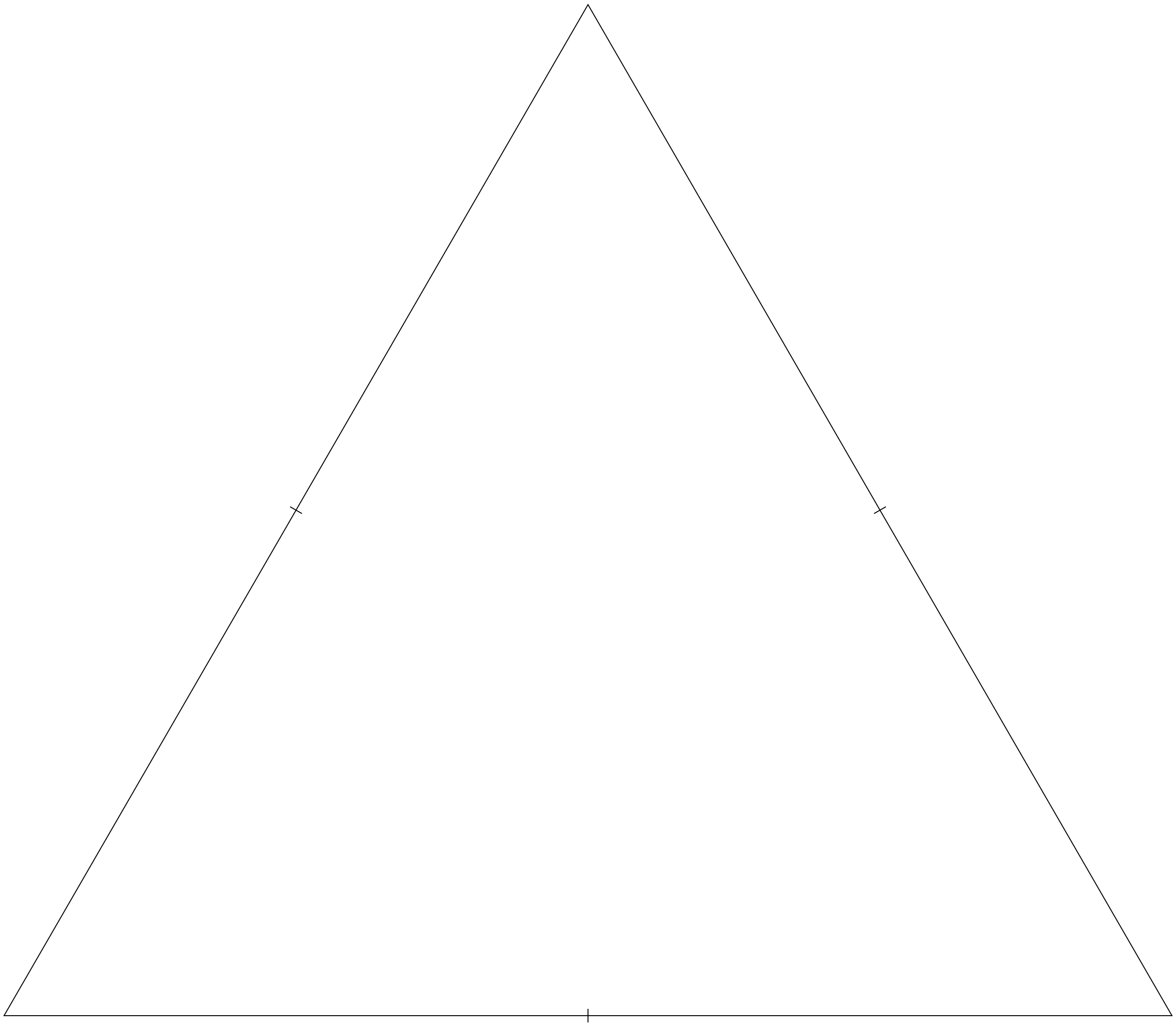
Make your own fractal

Do this page last. It's printed here to make it easier for you to use the rulers on the other side (when doing the following pages).

Start with a square □ or line — or something, and make a "rule" for what to change in each iteration; then keep repeatedly following your rule for a few iterations to see how it looks.

Sierpiński triangle

Instructions: each round (or "iteration," or "stage"), shade the middle of each  to make it look like  and then keep repeating.

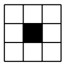


Questions:

F1. After one iteration, what fraction of the original triangle is shaded?
F2. After two iterations, what fraction of the original triangle is shaded?
F3. After three? Four? If you keep going forever, will the triangle ever get completely shaded?

M1. After one iteration, how many unshaded triangles $\triangle$ are there?
M2. After two iterations, how many unshaded triangles $\triangle$ are there?
M3. If there were N unshaded triangles in the last (previous) stage, then how many unshaded triangles will there be in this stage?
M4. After N iterations, how many unshaded triangles are there?

Sierpiński carpet

Instructions: each iteration, inside every square draw a tic-tac-toe board and shade the middle square so it looks like  and then keep repeating and repeating.

Questions (continued on next page):

F1. After one iteration, what fraction of the original square is shaded?
F2. After two iterations, what fraction of the original square is shaded?

F3. After three iterations? Four? If you keep going forever (infinite iterations), will it ever become totally shaded?

M1. Before the first iteration, there's one unshaded square $\square$; after the first iteration, how many unshaded squares $\square$ are there?
M2. After the second iteration, how many unshaded squares are there?
M3. After the third iteration, how many unshaded squares are there?
M4. If there are N unshaded squares after the last (previous) iteration, then how many unshaded squares will there be after this iteration?
M5. After N iterations, how many unshaded squares are there?

P1. Imagine the shaded parts are holes cut out of a carpet. If the original big square has perimeter 12, then what's the perimeter of the square that's cut out in the first iteration? Can you express that square's perimeter as a fraction of the original square's perimeter?
P2. In the second iteration, what's the sum of the perimeters of the eight squares you cut out? You can start by imagining the original square's perimeter is 81; then try to express your result as a fraction of the original square's perimeter.
P3. If the sum of the perimeters of the squares cut out in the last iteration is P, then what's the sum of the perimeters of the squares you cut out in this iteration?
P4. In iteration N, what's the sum of the perimeters of all the squares you cut out, as a fraction of the original square's perimeter?

Koch curve (note: “curve” in math does not mean curvy, just that you can draw it without lifting your pen)

Instructions: each iteration, replace every straight line segment with where all four line segments are the same length; and then keep repeating. Some examples: becomes and becomes

Questions:

L1. After one iteration, how many times longer is the “curve” than the original straight line? (Hint: if the original line is 3 units long, then how long does it become? Write this in terms of a fraction or multiple, not addition.)

L2. After two iterations, how many times longer is the curve than after the first iteration?

L3. If the curve is L units long in the last (previous) stage, then how long will it be in this stage?

L4. After N iterations, how many times longer is the curve than the original line?

A1. Consider an “equilateral” triangle △ (each side is the same length). How big is each inside angle? (Hint: the three angles inside any triangle sum to 180°.)

B1. Bonus question: the previous hint is only true in Euclidean geometry; show that you can draw a triangle on the surface of a sphere (like the Earth / a globe / an orange) with two 90° angles, or even three 90° angles(!). (I’d suggest putting the base on the equator.)

A2. Label each angle in the stage 1 curve: (hint: a straight line is a 180° angle)

A3. In the second stage curve, are any segments totally horizontal (parallel to the original line)? How can you prove it?

Fun fact: if you keep going forever (infinite iterations), the Koch curve you draw would not be one-dimensional (like a line), but also not two-dimensional (like a square); it has dimension $\frac{\log(4)}{\log(3)} = 1.26$! (The reason is a bit complicated, but if you imagine making everything on this paper twice as wide and twice as tall, then each one-dimensional line increase to two times the length and each square ◻ increases to four times the area ⊞ while each Koch curve gets approximately 2.4 times bigger.)

Koch snowflake

Instructions: same as for the Koch curve, just make sure all the “bumps” go on the outside. For example, in the first iteration, replace with and replace with and replace with