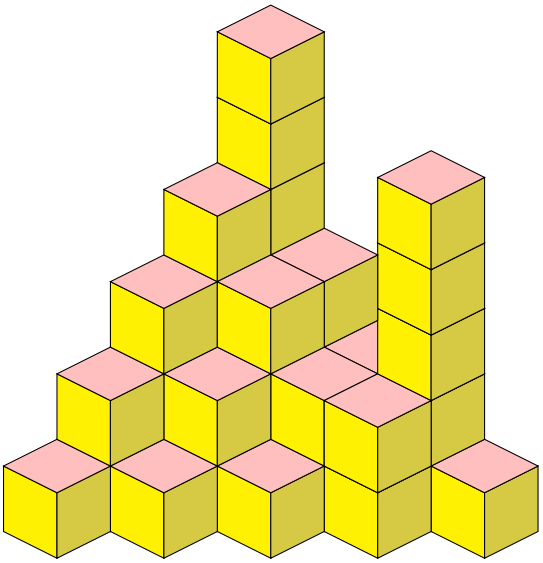
MOEMS Problems of the Week, 2022–24

9/5/22 How many cubes are in the below structure? Some cubes are not visible.

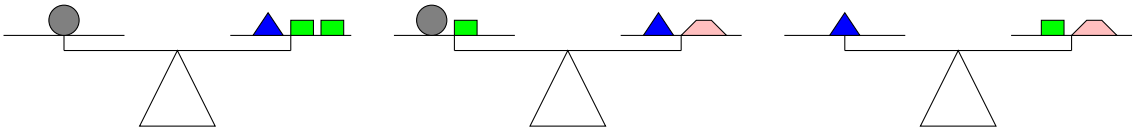


9/12/22 The complete outside (including the bottom) of a wooden 4-inch cube was painted purple. Then, it was cut into 1-inch cubes. How many of the 1-inch cubes do not have any purple paint on any face?

9/19/22 Suppose all the counting numbers are arranged in columns as show below. Under which column (letter) will 1000 appear?

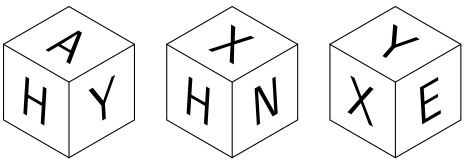
A	B	C	D	E	F	G
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	...	...

9/26/22 Each of the three diagrams below shows a balance of weights using different objects.



How many ▬ will balance a ●?

10/3/22 Below are three views of the same cube.



Which letter is on the face opposite H? Opposite X? Opposite Y?

10/10/22 A group of 21 people went to the county fair with 9 people on a stagecoach and 3 people in

each buggy. On the return trip, 4 people rode in each buggy. How many people returned on the stagecoach?

10/17/22 The perimeter of a rectangle is 22 inches, and the inch measure of each side is a counting number (1, 2, etc.). How many different areas in square inches could the rectangle have?

10/24/22 The product of two numbers is 504, and each of the two numbers is divisible by 6. However, neither of the two numbers is 6. What is the larger of the two numbers?

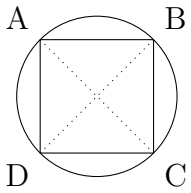
10/31/22 When Frankenstein, Dracula, and a mummy compared the amount of candy that they got on Halloween, they discovered that Frankenstein and Dracula together had 12 pieces, Dracula and the mummy together had 18, and Frankenstein and the mummy together had 10. Who has the least amount of candy, and how much is it?

11/7/22 ABCD is a square with diagonal AC of length 8 units. How many square units are in the area of the square? (Need to know Pythagorean theorem: given a "right triangle" with side lengths a, b, and hypotenuse c, $c^2 = a^2 + b^2$.)

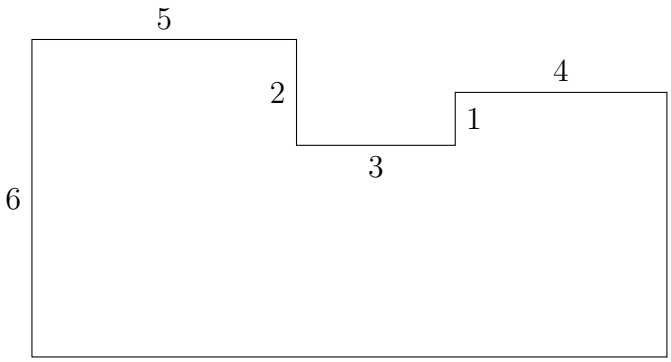
11/14/22 If you start with 4 and count by 3s, you get a sequence 4, 7, 10, ..., N. If N is the 15th number in the sequence, then what number does N represent?

11/21/22 What is the simplest form of the fraction $\dfrac{1}{1+\dfrac{1}{2+\frac{1}{3}}}$?

11/28/22 Square ABCD has all four of its vertices on a circle with diameter 10 units in length. Segments AC and BD are diagonals. How many square units of area does square ABCD have?



12/5/22 In the figure below, all corners are right angles, and each number represents the unit length of the segment nearest to it. How many square units of area does the figure have?



12/12/22 A jar filled with water weighs 10kg. When one half of the water is poured out, the jar and the remaining water weigh $5\frac{3}{4}$kg. How much does the jar weigh?

12/19/22 "Consecutive numbers" are whole numbers that follow in order, like $7, 8, 9, 10, 11, 12$. Find three consecutive numbers such that the sum of the first and the third is 118.

12/26/22 The average of six numbers is 7. When I remove two of the six numbers, the average of the remaining numbers is 8. What is the sum of the two numbers that I removed?

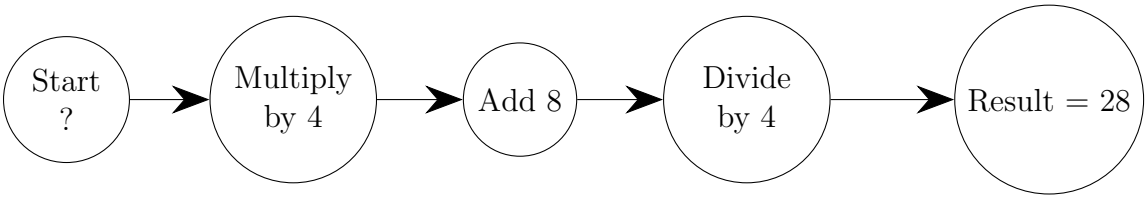
1/2/23 A4273B is a six-digit number in which A and B are digits, and the number is divisible by 72 with no remainder. Find the value of A and B.

1/9/23 A group of kids who like sticks made up their own words to keep count. "OC" means 8 sticks. "OCTA" means a bundle of 8 OCs, "OCTIL" means a bundle of 8 OCTAs, and "OCTILLA" means a bundle of 8 OCTILs. How many sticks are in an OCTILLA?

1/16/23 In the addition problem below, different letters stand for different digits. What five-digit number does SERVE represent?

$$\begin{array}{r} \ V\ C\ R \\ +\ V\ C\ C\ T \\ \hline S\ E\ R\ V\ E \end{array}$$

1/23/23 What should be the starting number in the diagram?

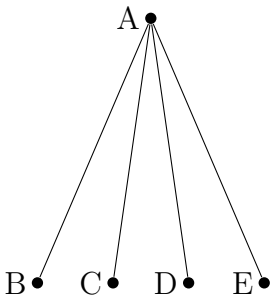


1/30/23 When certain numbers are placed in the empty boxes below, the sum is the same in each of the three rows, each of the three columns, and each of the two diagonals. What number should be in the center box? (Bonus: find all the missing numbers!)

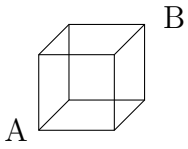
5		13
9	7	

2/13/23 Peter had a 12:00 noon appointment that was 60 miles from his home. He drove from his home at an average rate of 40 miles per hour and arrived 15 minutes late. At what time did Peter leave home for the appointment?

2/20/23 An acute angle is an angle whose measure is between 0 and 90 degrees (between 0 and $\pi/2$ radians). Using the line segments in the diagram, how many different angles can you find?

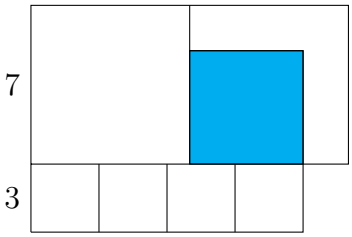


2/27/23 The length of the shortest trip from corner A to corner B along the edges of the cube is the length of three edges. How many different 3-edge routes are there from A to B?



3/6/23 I have exactly ten coins whose total value is $1. If three of the coins are quarters, then what are the remaining coins?

3/13/23 In the figure below, there are two large congruent squares with sides 7 units long, and four small congruent squares with sides of 3 units long. If the shaded figure is also a square, then what is its area in square units?



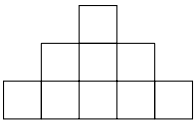
3/20/23 The month of January has 31 days. Suppose January 1 is a Monday. Then which day of the week is the 22nd day of the next month (February)?

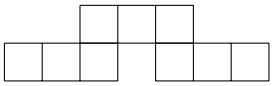
3/27/23 Suppose the average of 15 consecutive numbers is 15. What is the average of the first five numbers?

4/3/23 The owner of a bicycle store had a sale on bicycles (two wheels) and tricycles (three wheels). Each cycle has two pedals. When she counted the total number of pedals of the cycles, she got 50. When she counted the total number of wheels of the cycles, she got 64. How many tricycles were offered in the sale?

4/10/23 Person A was born on January 15, 1948. Person B was born on January 15, 1962. If both are alive now, in what year was Person A twice as old as Person B?

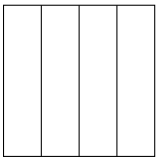
4/17/23 The small boxes in the two figures are all congruent squares. The perimeter of the first figure is 48cm. What is the perimeter of the second figure?





4/24/23 A train traveling 30km/hr reaches a tunnel that is 9 times as long as the train. If it takes 2 minutes from when the front of the train first enters the tunnel to when the back of the train exits the tunnel, then how long is the train?

5/1/23 The square below is divided into four congruent rectangles. The perimeter of each of the four congruent rectangles is 25 units. How many units are there in the perimeter of the square?

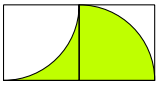


5/8/23 The sum of the weights of Tom and Bill is 138kg, and one boy is 34kg heavier than the other. How much does the heavier boy weigh?

5/15/23 Note: 1^2 means 1×1, $2^2 = 2 \times 2$, $3^2 = 3 \times 3$, etc. Also, it may help to know that $1^2 + 2^2 + 3^2 + 4^2 + \cdots + 25^2 = 5525$. Find the value of $N = 2^2 + 4^2 + 6^2 + 8^2 + \cdots + 50^2$.

5/22/23 A rectangular tile is 2cm by 3cm. What is the least number of tiles needed to completely cover a square region with side length 24cm?

5/29/23 ABCD and AFED are squares with a common side AD of length 10. Arc BD and arc DF are quarter-circles. What is the area of the shaded region?

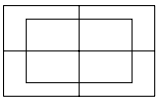


6/5/23 A square piece of paper is folded in half as shown and then cut into two rectangles along the fold. The perimeter of each of the two rectangles is 18cm. What is the perimeter of the original square?

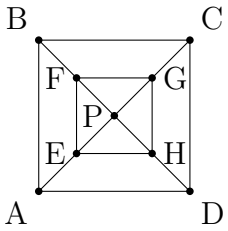
6/12/23 How many even numbers between 1 and 101 are multiples of 3?

6/19/23 In hoopball, a field goal is worth 2 points, and a foul shot is worth 1 point. Suppose a team scored 72 points and made 6 more field goals than foul shots. How many foul shots did the team make?

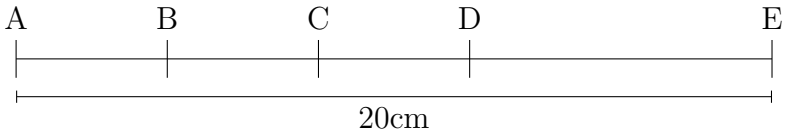
6/26/23 How many different rectangles can you find?



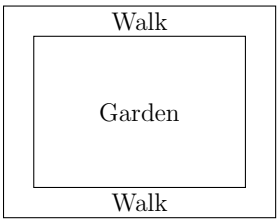
7/4/23 ABCD is a square; E, F, G, and H are midpoints of AP, BP, CP, and DP, respectively. What fractional part of the area of square ABCD is the area of square EFGH?



7/10/23 The length of AE is 20cm. B is the midpoint of AC, C is the midpoint of BD, and D is the midpoint of BE. What is the length of DE?



7/17/23 A rectangular garden is 5m by 7m and is bordered by a concrete walk 1m wide as shown. How many square fet are in the surface area of just the concrete walk?



7/24/23 Consider the property that when a number is divided by either 5 or 7, the remainder is 1. What is the smallest odd counting number larger than 10 that has this property?

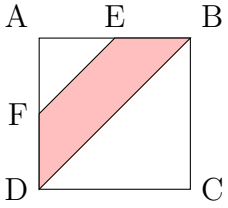
7/31/23 The product of two whole numbers is 10,000. If neither number contains a zero digit, then what are the two numbers?

8/7/23 If two days ago was Sunday, then what day of the week will 365 days from today be?

8/21/23 Six people participated in a checkers tournament. Each participant played exactly three games with each of the other participants. How many games were played in all?

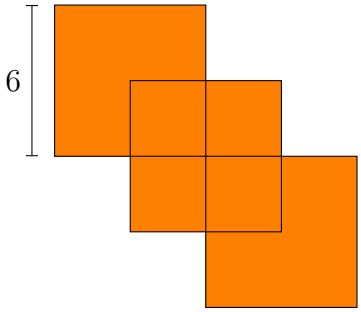
9/4/23 In a group of 30 high school students, 8 take French, 12 take Spanish, and 3 take both languages. How many students of the group take neither French nor Spanish?

9/11/23 ABCD is a square with area 16m^2. E and F are midpoints of sides AB and BC, respectively. What is the area of the trapezoid DFEB (shaded area)?



9/18/23 The age of a man is the same as his sister's age with the digits reversed. The sum of their ages is 99, and the man is 9 years older than his sister. How old is the man?

9/25/23 Three squares each have sides of length 6 units and overlap each other as shown. The points where the sides cross are midpoints. Find the area of the shaded figure.



10/2/23 Suppose all the counting numbers are arranged in columns as show below. Under which column (letter) will 300 appear?

A	B	C	D	E	F	G
1		2		3		4
	7		6		5	
8		9		10		11
	14		13		12	
15		16		...		...

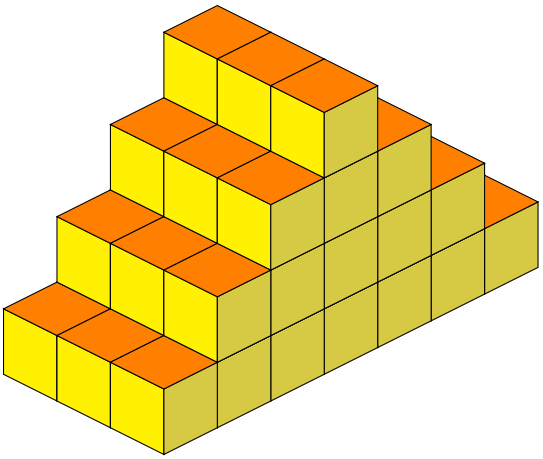
10/16/23 How many `x` markers are there in the figure?

```
xxxxxxxxxxxxxxxxxxx
xxxxxxxxxxx       xx
xxxxxxxxxxx       xx
xxxx xxxxxxx      xx
xxx   xxxxxxxxxxxxx
xx     xxxxxxxxxxxx
xxxxxxxxxxxxxxxxxxx
xxxxxxxxxxxxxxxxxxx
```

10/23/23 Which number makes the average of itself, 1/2, and 1/3 equal to one?

10/30/23 A "palimage" of a counting number has the same digits but in reverse order, like 659 and 956, or 1327 and 7231. Let X be the sum of 354 and its palimage. Add X to the palimage of X, and call the sum Y. What is the sum of Y and its palimage?

11/6/23 The block stairs in the figure were constructed by placing layers of cubes on top of each other. What is the total number of cubes contained in the staircase?



11/13/23 Four numbers are arranged in order of size, and the difference between any two adjacent numbers is the same. Suppose 1/3 is the first and 1/2 is the fourth: 1/3, __, __, 1/2. What are the two numbers between 1/3 and 1/2?

11/20/23 13 plums weigh as much as two apples and one pear. Four plums and one apple have the same weight as one pear. How many plums have the weight of one pear?

11/27/23 Cara, Cora, and Mia each have a different favorite sport among tennis, soccer, and baseball. Cara does not like baseball or soccer. Cora does not like baseball. Name the favorite sport of each person.

12/4/23 A restaurant has 30 tables, which are of two types. The first type seats two people, and the second type seats five people. A total of 81 people are seated when all seats are occupied. How many tables for two are there?

12/11/23 In the multiplication problem below, each blank space represents a missing digit. Find the product.

$$\begin{array}{r} 4\,\_\,\_ \\ \times\ \_\,7 \\ \hline \_\,\_\,8\,2 \\ 1\,2\,\_\,\_ \\ \hline \_\,\_\,\_\,\_\,\_ \end{array}$$

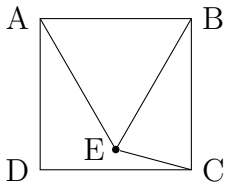
12/18/23 A box contains over 100 marbles. The marbles can be divided into equal shares among 6, 7, or 8 children with 1 marble left each time. What is the least number of marbles that the box can contain?

1/1/24 When 24 is added to a number, the result is the same as when the number is multiplied by 3. What is the number?

1/8/24 The angles of a triangle are in a ratio of 4:3:2. What is the degree measure of the second-largest angle? (Note: the angles of any triangle sum to 180°.)

1/15/24 The "tribonacci" sequence starts with terms $T_0 = T_1 = 0$ and $T_2 = 1$ and follows a pattern where each term is found by adding the three that came before it. For example, $T_3 = T_2 + T_1 + T_0 = 1 + 0 + 0 = 1$, and then $T_4 = T_3 + T_2 + T_1 = 1 + 1 + 0 = 2$. What is T_8?

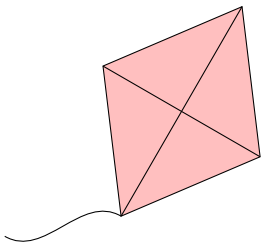
1/22/24 In the diagram, point E is drawn inside square ABCD such that triangle ABE is equilateral. What is the measure of angle BEC? (Note: the angles of any triangle sum to 180°.)



1/29/24 Ten years ago, Grayson was three times as old as Gabe. In five years, Grayson will be 10 years more than twice as old as Gabe. How old is Gabe right now?

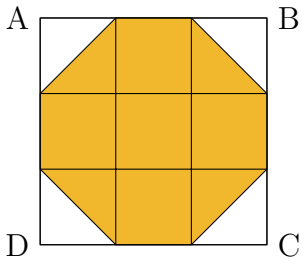
2/5/24 If you write out the first 20 odd counting numbers (1, 3, 5, ...), how many times does "3" appear as a digit?

2/12/24 The kite shown is a rhombus. The diagonals measure 16cm and 12cm. How many centimeters of ribbon are needed to line the perimeter of the kite? (Need to know Pythagorean theorem: for a "right triangle" with side lengths a, b, and hypotenuse c, $c^2 = a^2 + b^2$.)

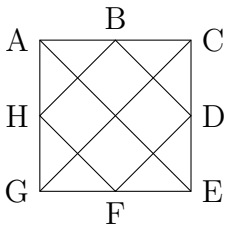


2/26/24 What is the value of the following, in simplest terms?
$(20 \times 24 \times 28 \times 32) \div (10 \times 12 \times 14 \times 16)$

3/11/24 Square ABCD is composed of nine congruent squares as shown. The area of the shaded region is 14cm^2. What is the area of square ABCD?



3/18/24 Square ACEG is drawn below. Points B, D, F, and H are midpoints of the sides of the square. How many squares can you find in this diagram?



4/1/24 On a standard 12-hour clock, the numerals 12 and 6 are opposite each other. On the planet Bajor, they use a circular 10-hour clock with the numerals 1 to 10 equally spaced. What pair of opposite numerals on a Bajorian clock has a sum of 11?

4/15/24 In a class of 26 students, 15 like vanilla ice cream and 16 like chocolate ice cream. Three students do not like either. How many students like both vanilla and chocolate ice cream?

4/22/24 Asher wants to buy 20 crayons. Toyworld sells crayons at 4 for 25 cents, and Gameland sells crayons at 5 for 30 cents. Which of the two stores sells 20 crayons for less, and by how much?

4/29/24 In the number 203,500, the last two zeros are called "terminal zeros." If $30 \times 40 \times 50 \times 60 \times 70$ is computed, how many terminal zeros would the product have?

5/13/24 Leo has 6 more pogs than Wilson. After Leo gives 10 pogs to Wilson, how many more pogs will Wilson have than Leo?

5/20/24 A rectangular box is 2cm high, 4cm wide, and 6cm deep. Elena packs the box with cubes, each 2cm by 2cm by 2cm, with no space left over. How many cubes does she fit in the box?

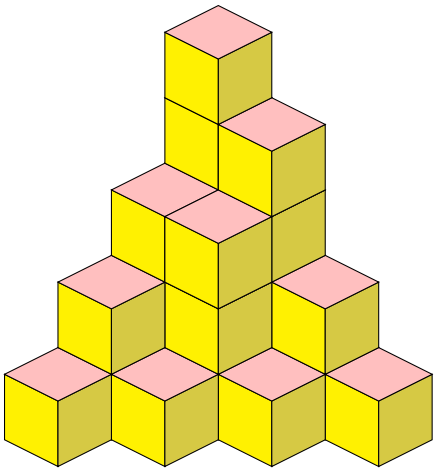
5/27/24 Below, each blank represents a digit, and different letters represent different non-zero digits. Which three-digit number is the least possible product?

$$\begin{array}{r} A\ B \\ \times\ C\ B \\ \hline \_\ 9 \\ \_\ \_\ \\ \hline \_\ \_\ \_ \end{array}$$

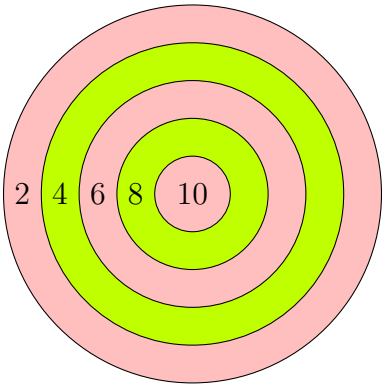
6/10/24 Admission to the local movie theater is \$3 for each child and \$7 for each adult. A group of 12 people pay \$64 total for admission. How many children are in this group?

6/17/24 Suppose a standard 12-hour clock now shows a time of 10:45. What time will the clock show 100 hours from now?

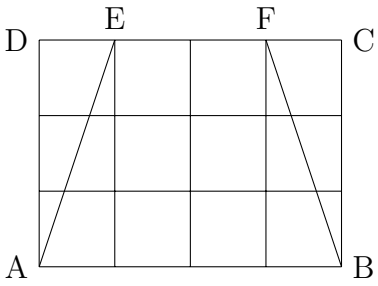
6/24/24 The block tower shown is made by placing congruent cubes on top of each other with no gaps. Not all cubes are visible. How many cubes does the tower contain?



9/9/24 Reuben throws 5 darts at the target shown. Each dart lands in a region of the target, scoring the points shown. Of the following total scores, list all that are *not* possible: 6, 14, 17, 38, 42, 58.



9/16/24 ABCD is a rectangle whose area is 12 square units. How many square units are contained in the area of trapezoid EFBA?



9/23/24 What is the value of 1/2 of 2/3 of 3/4 of 4/5 of 100?

9/30/24 Melody was elected class president. She received 3 votes for every 2 votes that Gerald got. No one else ran. However, if 8 people who voted for Melody had instead voted for Gerald, then Melody would have received only one vote for every two that Gerald would have got. How many people voted?

10/7/24 The area of a square is 36cm^2. A rectangle has the same perimeter as the square. The length of the rectangle is twice its width. What is the area of the rectangle?