July OYMC Individual Examination

Elementary Division

OYMC Committee

July 17, 2021

1 Instructions

Talking about the test before July 18th is strictly forbidden, as well as having outside help through the internet or interaction through people. Actions like these will result in disqualification and possible banning from ever taking the test again.

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The answer to every problem will be a nonnegative integer x between 0000 and 9999, inclusive. No leading zeros are necessary. Some questions will have *choices* - write the number corresponding to the correct choice in that case.

The Individual Round is 45 Minutes long. You have 40 minutes to solve the questions and 5 minutes to submit your answers. At the end of the testing period, the form will close and no new submissions will be accepted. Only pencils, blank paper, and rulers are allowed on the exam. Calculators and protractors are prohibited. Your test will start on the next page. Good luck!

Remark 1.1. **Note:** 2 positive integers are relatively prime if their greatest common divisor is 1.

2 Problems

Problem 1. Evaluate $(2+0+2+1)(2+0+2-1)$.

Problem 2. Determine the total number of (not necessarily distinct) letters in the block below.

O O O O
Y Y Y Y
M M M M
C C C C

Problem 3. Richard has baked anywhere from 1 to 20 cookies. A Blob claims there are less than 15 cookies while another Blob claims there are more than 15 cookies. If all Blobs always lie, how may cookies are there?

Problem 4. A square piece of paper is folded exactly in half twice along the purple lines as shown. The new resulting figure has an area of 9m^2. What was the area of the original square?

Problem 5. In the following equation: $5 \cdot \frac{3}{7-4} = (2+4)\Delta 1$, there is a missing operation symbol replaced with a "Δ". If this operation symbol is among the four basic operations: Adding, Subtracting, Multiplying, and Dividing, which operation is it?

(1) $+$ **(2)** $-$ **(3)** $\times$ **(4)** $\div$

> *Remark 2.1.* **Note:** Please answer with the integer written to the left of the operation you believe is correct.

Problem 6. Consider the following pattern:

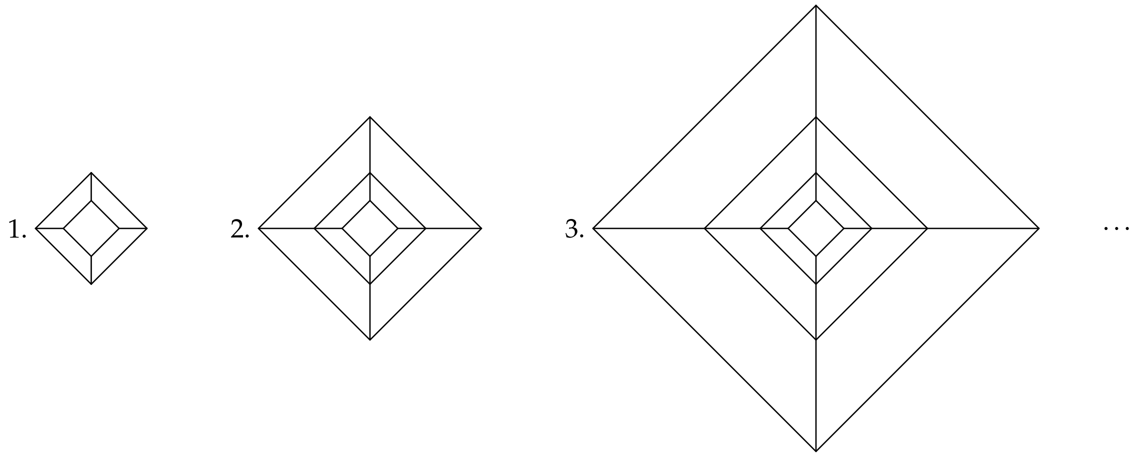


Figure 1 contains 5 regions. Figure 2 contains 9 regions. Figure 3 contains 13 regions. If this pattern continues, how many regions will there be in figure 12?

Problem 7. Oliver has written the sequence $1, 2, 3, \ldots, 99$ on a blackboard. Because his friend Ben has done the exact same thing, Oliver decides to change his sequence by adding a negative sign in front of all even integers in his sequence. If the positive difference between the sum of the Oliver's and Ben's sequences is S, what is $S \div 100$?

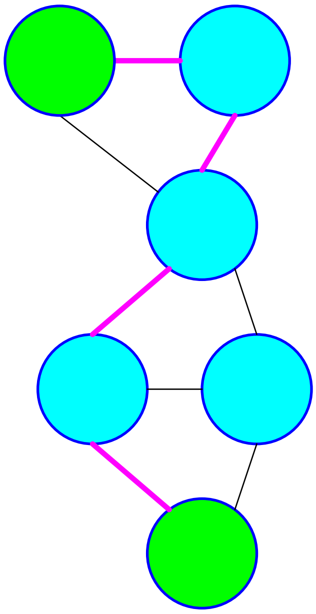
Problem 8. On any given day, Nolan randomly chooses to: read a book, eat a snack, or do math problems. Unfortunately, he also enjoys running away from home. Nolan runs away:

- 90% of the time when he reads a book;
- 30% of the time when he eats snacks;
- 10% of the time when he does math problems.

Suppose Nolan's mom is happy if and only if he: reads and doesn't run away *or* does math problems and doesn't run away. If the probability Nolan's mom becomes happy on a random day can be expressed as $\frac{a}{b}$ where a and b are relatively prime positive integers, determine $a + b$.

Problem 9. At an Easter Egg Hunt, contestants run around and attempt to find as many eggs as possible. Each egg is worth 1 point, but the golden egg is worth 10 points. There exists only one golden egg. At the end, Mark says, "I have twice as many eggs as Sarah," while Sarah says, "I have twice as many points as Mark." How many eggs does Sarah have?

Problem 10. In the game of *Circle Jump*, one can only jump from one circle to another if there is a line connecting the two circles. How many paths consisting of 4 jumps began at one of the green circles and end on the other green circle? (One possible path has been highlighted in purple.)

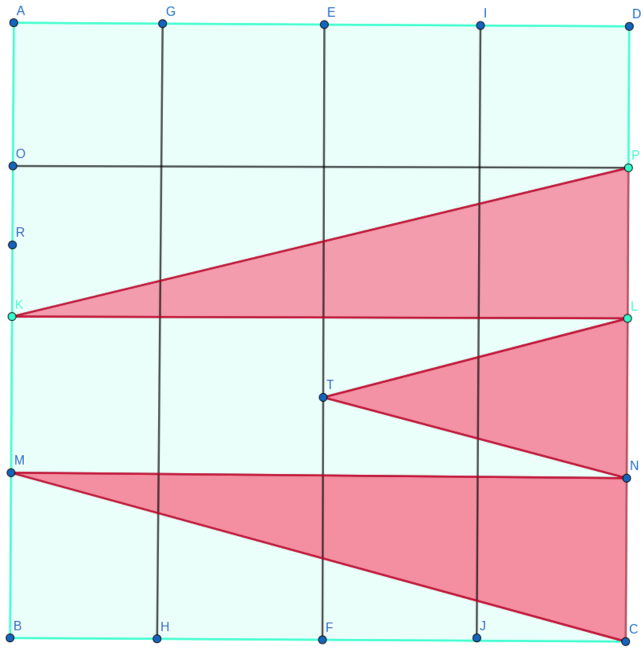


Problem 11. Sam needs to drive from his home to the local gas station in 8 minutes. If he drives at a rate of 20 miles per hour, he'll be exactly 2 minutes late. How fast should he drive, in miles per hour, in order to arrive at the local gas station exactly on time?

Problem 12. How many 2-digit numbers AB (where A and B represent the tens and units digit respectively and A is nonzero) are there such that when the digits of the numbers are flipped giving the number BA, the difference $AB - BA$ is a multiple of 9?

Problem 13. Jerry is filling barrels with salt. If he adds three bags of salt, then the barrel weighs 315 kg. If he adds 5 bags of salt, the barrel weighs 355 kg. How many bags must Jerry add to the barrel if he wants it to weigh at least 1000 kg?

Problem 14. The diagram below shows a 4 by 4 square that has been split into 16 congruent 1 by 1 squares. If the ratio of the area of the red region to the area of the entire square can be expressed as $\frac{p}{q}$ for relatively prime positive integers p and q, find $p+q$.



Problem 15. Alex, Bob, Caroline, Daniel, Evelyn, and Fiona are playing in a tennis tournament against one another. In the first round, Alex plays Bob, Caroline plays Daniel, and Evelyn plays Fiona. In the second round, all the winners of the first round are randomly paired up with each other while an extra winner who isn't able to be paired with anyone is paired up with a player that is randomly chosen from the pool of people who lost in the first round. The winners of these matches go on to face each other in a final match to determine the winner of the tournament. In how many ways can the tournament be arranged?

Problem 16. In my school, 60% of the students have a phone, and 75% of the students who have a phone wear glasses. Additionally, 50% of the overall students at my school don't wear glasses. What percentage of students at my school don't have a phone and don't wear glasses?

Problem 17. Jared prides himself in speed reading, and can read 20 pages per minute, while his mom can read 10 pages per minute. Jared and his mom compete to see who can complete *Prealgebra* by AoPS faster. They both begin at the exact same time but when Jared has 10 more pages to go, he decides to take a break. The moment Jared starts reading again, his mom finishes reading the book. If the book is 610 pages, how many minutes long was Jared's break?

Problem 18. Consider regular hexagon $ABCDEF$ such that AC and BF intersect at X. If $AC = 12$, then the length of XD can be expressed as $a\sqrt{b}$ in simplest radical form. What is $a + b$?

Problem 19. Determine the largest number of primes that could be contained in a set of 10 consecutive positive integers.

Problem 20. Consider triangle ABC such that $AB = 15, BC = 20$, and $AC = 25$. Let D be the foot of the altitude from B to AC, and let circle C_1 have diameter BD. If C_1 intersects AB at X and BC at Y, determine the length of XY.

Problem 21. Every morning, Jeremy has to perform the following 6 tasks:

- Brush his teeth;
- Eat Breakfast;
- Change into school clothes;
- Make his bed;
- Let his dog outside;
- Pack up his backpack.

Suppose Jeremy:

- Must change into his school clothes before he lets his dog outside;
- Has to eat breakfast and make his bed consecutively (but not necessarily in that order).

Determine the number of ways Jeremy can rearrange the ordering of his 6 tasks to complete each day.

Problem 22. Let $ABCD$ be a trapezoid with bases AB and CD such that $AD = BC = 2\sqrt{3}$. The circle centered at A with radius AB intersects CD at E. Given that E is the midpoint of CD, the area of the trapezoid that is outside of the circle can be expressed as $a\sqrt{b} + c \cdot \pi$, where a, b, c are positive integers and b is square-free. Find $a + b + c$.

Problem 23. If each letter in the equation

$$\begin{array}{ccccc} & \mathrm{T} & \mathrm{H} & \mathrm{I} & \mathrm{S} \\ + & \mathrm{V} & \mathrm{E} & \mathrm{R} & \mathrm{Y} \\ \hline & \mathrm{H} & \mathrm{A} & \mathrm{R} & \mathrm{D} \end{array}$$

represents a distinct, positive, single-digit integer, what is the largest possible value of $\overline{HARD}$?

Problem 24. How many ordered triplets of positive integers (a, b, c) satisfy

$$a^2 + b^2 + c^2 \leq 2a + 2b + 2c + 2?$$

Problem 25. Let a and b be (not necessarily distinct) two-digit positive integers selected at random. If $\frac{m}{n}$ is the probability that adding a and b and subtracting $\min(a, b)$ from $\max(a, b)$ both don't require carrying (in base 10) for relatively prime positive integers m and n, compute $m + n$.

> *Remark 2.2.* **Note:** Carrying is mathematically defined as $a \pmod{10} + b \pmod{10} > 10$, where $x \pmod{10}$ is the remainder when x is divided by 10.

> *Remark 2.3.* **Note:** $\max(a, b)$ is the larger element of $\{a, b\}$, while $\min(a, b)$ is the smaller element of $\{a, b\}$. (When $a = b$, $\max(a, b) = \min(a, b) = a = b$.)