# Strategies and Terms (12220)


## Fairview Elementary Math Club

https://kaplandm.github.io/FVE/

# Types of Questions

1. You either know it, or you don't (and should skip)
   - $0.\overline{6} =$ _____ (fraction)
   - The measure of an interior angle in an equilateral triangle is _____°
2. You need to know some term/notation, and then do work
   - The number 66 written in *base 4* is _____$_4$
   - The *perimeter* of a *regular octagon* with a side length of 14 is _____
3. Slow way, fast way, faster way
   - The sum of the terms of the arithmetic sequence $4, 8, 12, \ldots, 40$ is _____
4. (Sprint) Can eliminate options (and guess)
   - Jeremiah spends \$2.50 to play five rounds of an arcade game. At the same price per arcade round, how much would seven rounds cost?
     (A) \$2.50 (B) \$0.50 (C) \$2.10 (D) \$3.50 (E) \$4.20

# Grouping

Number Sense 18: $18 + 24 + 30 + 36 + 42$ (pair to make multiples of 10)

- 

NS 21: $25 \times 28$ (what's easy to multiply 25 by?)

- 

NS 32: $16 \times 99$ (what's near 99 but easier to multiply by?)

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Other examples

- NS 5, 8, 15, 19, 24, 28, 30
- Sprint 2
- Target 1
- Team 6 (geometric grouping)

# Grouping

Number Sense 18: $18 + 24 + 30 + 36 + 42$ (pair to make multiples of 10)

- $(18 + 42) + (24 + 36) + 30 =$

NS 21: $25 \times 28$ (what's easy to multiply 25 by?)

- $25 \times 4 \times 7 =$

NS 32: $16 \times 99$ (what's near 99 but easier to multiply by?)

- $16 \times (100 - 1) = 16 \times 100 - 16 \times 1 =$

Other examples

- NS 5, 8, 15, 19, 24, 28, 30
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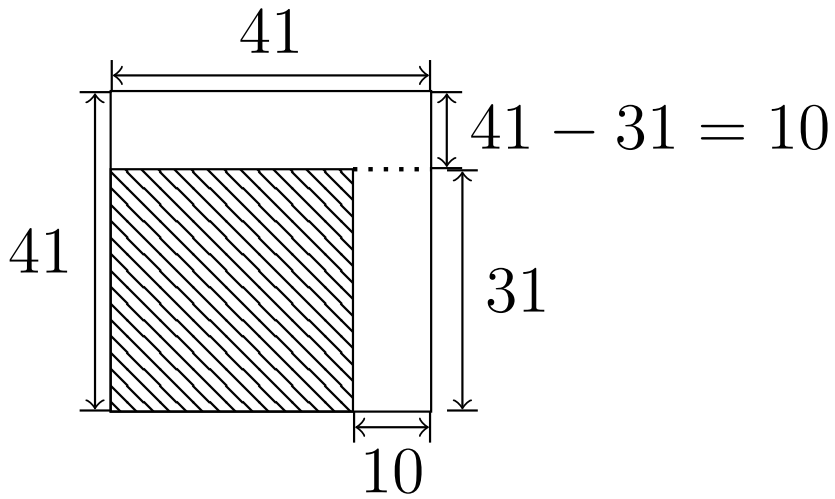
# Squares

NS 12: $12^2 =$ _____? This means "12 squared," or $12 \times 12$

NS 46: $41^2 - 31^2 =$ _____? The $41^2$ is the area of a square with side length 41, so draw $41^2 - 31^2$ as




NS 25: $22^2 =$ _____

Other examples: NS 35, 68

# Divisors and GCD

Divisor: a divisor of number $N$ is a whole number that divides into $N$ with no remainder (so 1 and $N$ are always divisors)

- The divisors of 8 are 1, 2, and 4; 3 is not a divisor because $8 \div 3$ has remainder 2

GCD (greatest common divisor): the GCD of two numbers $X$ and $Y$ is the largest whole number that's a divisor of both $X$ and $Y$

- Ex: divisors of 6 are $1, 2, 3, 6$; divisors of 10 are $1, 2, 5, 10$; common divisors are $1, 2$; GCD is 2
- NS 22: The GCD of 8 and 18 is _____

Other examples

- NS 71, 73
- Sprint 19

# Powers (Exponents)

Notation: $n^p = \overbrace{n \times n \times \cdots \times n}^{p \text{ times}}$

- $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

NS 48: $3^5 =$ _____

NS 55: $9^3 =$ _____

Other examples: NS 70, 80

Also: $n^0 = 1$ (if $n \neq 0$)

- $2^0 = 3^0 = 100^0 = 0.334^0 = 1$

Also: $n^{-p} = \frac{1}{n^p}$ (so $n^{-1} = \frac{1}{n}$, $n^{-2} = \frac{1}{n^2}$, etc.)

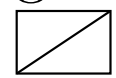
- $2^{-1} = \frac{1}{2^1} = \frac{1}{2}$, $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$, etc.
- NS 74: $2^x = \frac{1}{2}$, so $x = -1$

# Triangles

The three inside angles add up to 180°

- Ex: if first two angles are 160°and 10°, then the third must be $180 - 160 - 10 = 10$ degrees
- If all angles equal, then each is 60° (equilateral $\triangle$)

"Right" triangle: one angle is 90°, longest leg = hypotenuse

- Half rectangle; area is half of rectangle's: 
- Right triangle if leg length ratio 3:4:5

NS 56: The measure of an interior angle in an equilateral triangle is _____°

NS 66: The area of a right triangle with a leg of length 6 and a hypotenuse of length 10 is _____

Other examples: Sprint 30, Target 6

# Fraction/Decimal/Percent Conversion

Percent is special fraction with denominator 100

- Ex: $\frac{30}{100} = 30\%$, $99\% = \frac{99}{100}$

Decimal shows tenths, hundredths, etc.

- Ex: $0.2 = \frac{2}{10}$, $0.23 = \frac{2}{10} + \frac{3}{100}$

Helpful to memorize some common ones

- $1/2 = 0.5 = 50\%$
- $1/3 = 0.\overline{333} = 33\frac{1}{3}\%$, $\frac{2}{3} = 0.\overline{666}$
- $1/4 = 0.25 = 25\%$, $\frac{3}{4} = 0.75$
- $1/5 = 0.2 = 20\%$
- $1/6 = 0.1\overline{666} = 16\frac{2}{3}\%$
- $1/7 = 0.\overline{142857}$, $\frac{2}{7} = 0.\overline{285714}$
- $1/8 = 0.125 = 12.5\%$, $\frac{3}{8} = 0.375$, $\frac{5}{8} = 0.625$, $\frac{7}{8} = 0.875$
- $1/9 = 0.\overline{111} = 11\frac{1}{9}\%$
- $1/10 = 0.1 = 10\%$

# Fraction/Decimal/Percent Conversion

NS 31: $\frac{1}{2} =$ _____% (have memorized)

NS 64: $0.\overline{6} =$ _____ (fraction) (have memorized)

NS 34: 10% of 120 is _____

- Use $10\% = \frac{1}{10}$

NS 27: The greater of $\frac{3}{5}$ and $\frac{5}{8}$ is _____?

- Fast way: if memorized, then immediately $\frac{5}{8} = 0.625$ greater than $\frac{3}{5} = 0.6$

NS 50: (estimate) $539 \times 333$

- Use $\frac{1}{3} \approx 0.333$, so $333 \approx \frac{1}{3} \times 1000$. Combine with $539 \approx 540 = 10 \times 54$ to get $539 \times 333 \approx 10 \times (54 \times \frac{1}{3}) \times 1000$

NS 60: (estimate) $142857 \times 14$ (see $142857 \approx 1{,}000{,}000 \times \frac{1}{7}$)

# Bases

We usually use base 10

- Column values (right to left): $1, 10, 100, \ldots$
- Ex: 154 means $4 \times 1 + 5 \times 10 + 1 \times 100$

Base $n$ column values are $1, n, n^2, n^3, \ldots$

- Base 2 (binary): $1, 2, 4, 8, \ldots$;
  $1011_2 = 1 \times 1 + 1 \times 2 + 0 \times 4 + 1 \times 8 = 11_{10}$
- Base 8 (octal): columns $1, 8, 64, \ldots$;
  $123_8 = 3 \times 1 + 2 \times 8 + 1 \times 64 = 83_{10}$

NS 49: $63_9$ in base 10 is _____

- Base 9 columns are $1, 9$, so $63_9 = 3 \times 1 + 6 \times 9$

NS 69: The number 66 written in base 4 is _____$_4$

NS 54: $110001_2$ in base 8 is _____$_8$

# Dice and Coins (Probability)

At this level: usually all possible outcomes are equally likely, so write them out and count how many satisfy the condition

- Coin ex: flip two fair coins, what's probability of both heads? Four possible outcomes: TT, TH, HT, HH. One satisfies condition (both heads). So probability is 1/4.
- Dice ex: roll two dice, what's probability sum is 7?

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

- NS 61: Two fair dice are rolled. The probability the sum of the numbers shown is 3 is _____ (fraction)
- Target 8

# What is the final question asking?

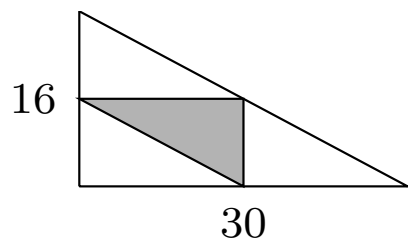
Sprint 7: This past week, Ellen spent 15 minutes starting an essay on Monday. Each day after Monday, she spent 15 more minutes working on the essay than she did the day before. After she finished her essay on Thursday, how many total minutes had she spent doing the essay?
"(B) 60" is the Thursday minutes, trying to catch you!

# Multiple Choice: Educated Guessing

Sprint 5: Jeremiah spends \$2.50 to play five rounds of an arcade game. At the same price per arcade round, how much would seven rounds cost?
(A) \$2.50 (B) \$0.50 (C) \$2.10 (D) \$3.50 (E) \$4.20

Sprint 9: A whole number greater than 680 and less than 687 is a multiple of 4. What is the units digit of that number? (A) 6 (B) 0 (C) 4 (D) 8 (E) 2

Sprint 16: The right triangle shown below has legs of length 16 and 30. The smaller gray triangle is formed by connecting the midpoints of the right triangle. What fraction of the total area is not gray?
(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$



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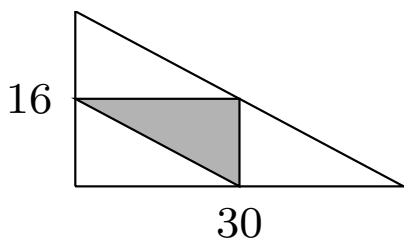

# Guess and Check

Sometimes you can check if a guessed answer is too low or too high (or correct)

Target 2: A four-digit number is odd, a multiple of 5, and a multiple of 11. The hundreds digit of the numbers is a 3, and the thousands digit is the same value as the tens digit. What is the four-digit number?

- Odd multiple of 5 $\implies$ ones digit is 5
- Hundreds digit is 3 $\implies$ D3D5, what's D?
- Guess, then divide by 11 to check; ex: $1315 \div 11$? (Remove 1100; $215 \div 11$? No, because $220 = 11 \times 20$.) Try $2325 \div 11 \ldots$

Target 4: Silver studs are worth 10 points and gold studs are worth 100 points. Chase has 5 more silver studs than gold studs, and the total value of Chase's silver and gold studs is 1700 points. What is the total number of silver and gold studs that Chase has?

- If all gold $\implies$ 17 gold, so guess $16, 15, \ldots$
- Check if 16 gold: $1600 + 21 \times 10 = 1810$, too high

# Other Terms and Ideas

Mode: the most frequent number in a list

- NS 58: The mode of the list $1, 2, 2, 3, 3, 3, 4$ is _____

Mean (average): sum of numbers divided by how many

- Mean of $1, 2, 3$ is $\frac{1+2+3}{3} = 2$
- Mean of $1, 2, 3, 6, 9$ is $\frac{1+2+3+6+8}{5} = 4$

Multiples, LCM (least common multiple)

- NS 38, Target 5

Divisibility by...

- 3: digits sum to multiple of 3? 24: $2 + 4 = 6$, yes. 187: $1 + 8 + 7 = 16$, no.
- 9: digits sum to multiple of 9? 72: $7 + 2 = 9$, yes. 730: $7 + 3 + 0 = 10$, no.
- 5: ones digit is 5 or 0

# Other Terms and Ideas (con't)

Square root: $\sqrt{n} = r$ means $r^2 = n$

- Square of area $A$ has side length $\sqrt{A}$; area 9, side $\sqrt{9} = 3$
- $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$
- NS 57: $\sqrt{2116} = \sqrt{4 \times 529} = \sqrt{4} \times \sqrt{529} = 2 \times \sqrt{529}$. Then guess/check: $20^2 = 400$, $25^2 = 625$, ends in 9...
- NS 75: $\sqrt{18} \times \sqrt{8} = \sqrt{18 \times 8} = \sqrt{9 \times 2 \times 2 \times 4} = \sqrt{9} \times \sqrt{4} \times \sqrt{4}$
- (Bonus: what's $\sqrt{-1}$??)

Permutations: number of ways to order different objects

- If $n$ different objects, then $n \times (n-1) \times \cdots \times 2 \times 1$
- Often complicated by other conditions (Sprint 14)

Pythagorean theorem: right triangle with side lengths $a$, $b$, and $h$ (hypotenuse) always has $a^2 + b^2 = h^2$ (whoa!)

- Most common: $(a, b, h) = (3, 4, 5)$, or some multiple like $(6, 8, 10)$; or $(5, 12, 13)$ or its multiples
- NS 66, Sprint 22