Math League 12210 (middle school), selected questions

Sprint 1 _____ A box of chocolates has 4 rows containing 6 chocolates each. Anshul takes them all out, eats three of them, and splits the remaining chocolates into 3 equal groups. How many chocolates does each group have?

Sprint 2 _____ John draws a rectangle with side lengths 7 and 6. What is the numerical difference between the area and perimeter of John's rectangle?

Sprint 3 _____ Maggie reads two comic books on Monday. Starting on Tuesday, she reads twice as many comic books as she does the previous day. How many comic books does she read from Monday to Friday of the same week?

Sprint 5 _____ Elvis rolls a fair six-sided die. What is the probability that the die shows a number that is not prime? Express your answer as a common fraction.

Sprint 8 _____ Kaycee is running a lemonade stand, where she sells one lemonade per person. Each lemonade costs \$1 to make, and Kaycee sells lemonade for \$1.20. Kaycee estimates that 500 people will buy lemonade one hot day, but the actual number of people who bought the lemonade was 25% more than Kaycee's estimate. If Kaycee makes exactly as many lemonade as she sells, how much profit, in dollars, does Kaycee make that day?

Sprint 10 _____ What is the positive difference between the minimum and maximum area of a rectangle with whole number side lengths and perimeter 44?

Sprint 13 _____ At an intramural competition, Team Power, Team Wisdom, and Team Courage each have 6 people. Brandon is a leader who plans on selecting a number of people at random to do the master trial. If he wants to guarantee that at least one person from each team participates, how many people must Brandon select at random? (DMK: I find it easier to think about the related question, "What's the highest number of people he could select and still have nobody from the third team?")

Sprint 15 _____ One week from Monday to Friday inclusive, Sam does some number of pushups in the martial arts dojo. On Monday and Wednesday, Sam does 8 pushups. On Tuesday, Sam does 12

pushups. On Thursday, Sam does a whole number of pushups, and on Friday, Sam also does a whole number of pushups. After Friday, Sam calculated the median number of pushups he did over the past five days. What is the sum of all possible values of Sam's result? (DMK: the median is the middle number after you sort the five numbers from low to high.)

Sprint 18 (modified) _____ What is the number of different arrangements of the letters of the word VIVID?

Sprint 20 (modified) _____ A hexagon has two opposite vertices colored white and blue, and all other vertices are colored gray. A move consists of swapping the colors of the white vertex and a vertex adjacent to it by an edge. At most how many moves are required in order to swap the positions of the white and blue vertices?

Sprint 24 _____ One highway has 900 mile markers in a single-file line on the road such that the distance between two consecutive mile markers on the road is 1 mile. Daniel wants to install the lowest number of call boxes such that each mile marker is at most one mile from a call box. What is the lowest number of call boxes that Daniel needs to install?

Sprint 27 (modified) _____ Using only the uppercase Greek letters Γ, Δ, Π, Σ, Φ, and Ψ (whose respective names are gamma, delta, pi, sigma, phi, and psi), Elise tries to come up with a name for her college club by writing down all sequences of 3 letters that contain exactly two different letters, like $\Pi\Phi\Pi$ or $\Delta\Delta\Gamma$. How many different sequences can Elise write?

Target 1 (modified) _____ What is the sum of all positive even whole numbers that are less than 19?

Target 3 (modified) _____ Evan goes to bed at exactly 9:55pm and wakes up at exactly 6:01am the next day. The minimum recommended number of hours to sleep for young adults his age is 7 hours. How many more minutes did Evan sleep compared to the minimum recommended number of sleep hours for young adults?

Target 4 _____ Becca and Madison are playing a game during their lunch break. Becca flips four fair coins and wins if the number of heads is greater than the number of tails. What is the probability

that Becca wins the game? Express your answer as a common fraction. (DMK hint: start by writing out all the possible outcomes of the coins in the order they're flipped, like HHTH or TTTH; how many are there, and how many let Becca win?)

Target 7 _____ A line that is perpendicular to the line $y = 2x + 3$ is drawn. The lines intersect at a point with a y-coordinate of 3. What is the sum of the coordinates of the y-intercept of the line that was drawn?

Team 4 (modified) _____ The "half-life" of an object is the amount of time it takes for that object's mass to decrease by half. Collin finds a rock that has a half-life of 1 year. To the nearest whole number, how many years will it take for the rock to reach 3.1% of its original mass?

Team 6 _____ A list of four positive whole numbers is created. For each pair of numbers in the list, the larger of the two numbers is written down. If the six numbers that are written down are 1, 2, 2, 5, 5, and 5, then what is the sum of the four numbers in the original list? (DMK: note "positive" does not include 0, only 1, 2, 3, etc.)

Team 8 (modified) _____ Andrea, Belinda, Chelina, and Diana are standing in a line for a group photo. Andrea is not next to Diana and Chelina is not next to Belinda. How many different ways can the four people be standing in a line?

Countdown 2 _____ What is the fifth triangular number?

Countdown 3 _____ The measures of the angles of a triangle are in the ratio $2 : 3 : 4$. What is the degree measure of the smallest angle in the triangle?

Countdown 4 _____ Simplify: $-1 + 2 - 3 + 4 - 5 + 6 + \cdots + 2012$.

Countdown 7 _____ A fair 6-sided die is rolled three times. Find the probability that the values of the rolls are strictly increasing. Express your answer as a common fraction

Countdown 10 _____ What is the base-16 number $4A_{16}$ in base 10? (The base 16 digits in order

are $0, 1, 2, 3, \ldots, 9, A, B, C, D, E, F$.)

Countdown 12 _____ If x, y, and z are positive integers such that $xyz = 120$, what is the least possible value of $x + y + z$? (DMK: this is related to questions about minimum perimeter given a certain area.)

Countdown 20 _____ Evaluate: $3 + 6 + 9 + \cdots + 33$.

Countdown 26 _____ Three of the faces of a rectangular prism have areas 20, 24, and 30. What is half of its volume?

Countdown 27 (modified) _____ Three mechanics can fix two cars in four days. Assuming all mechanics work at the same rate, how many cars can five mechanics fix in six days?

Countdown 36 _____ In Japan, there are coins worth 1 yen, coins worth 5 yen, and coins worth 10 yen. How many different combinations involving any number (or none) of these three coin types are worth 25 yen? Two combinations are considered identical if they consist of the same number of coins for all three coin types.

Countdown 42 (modified) _____ Evaluate: 111^2.

Countdown 46 _____ Lucas is buying boxes of chicken nuggets, where each box either has 5 nuggets or 8 nuggets. What is the maximum number of nuggets that Lucas couldn't get in exact amount from buying some number of boxes that can be either size?

Countdown 47 (modified) _____ What is the positive difference between the sum of the first 20 positive integers and the sum of the first 20 positive odd integers?

Countdown 50 (modified) _____ Riley is picking 2 people from a soccer team of 7 people to have a role of defender. If the order Riley assigns defenders does not matter, how many ways can Riley pick the defenders?

Countdown 51 _____ Simplify: 72×11.

Countdown 53 _____ What is the sum of all integers that are one more than a one-digit prime?

Countdown 56 _____ Find 6^3.

Countdown 61 _____ The side lengths of a rectangle are both integers. If the perimeter of the rectangle is 18, what is the smallest possible value for the area of the rectangle?

Countdown 67 _____ What is the positive difference between the largest 3-digit number and the smallest 3-digit number?

Countdown 76 _____ What is 20% of 60% of 500?

Countdown 80 _____ How many times does $\frac{1}{49}$ go into $\frac{2}{7}$?

Math League 12310 (middle school), selected questions

Sprint 1 _____ What is the greatest number of right interior angles that a triangle can have?

Sprint 5 _____ A triangle and a square have the same perimeter and share a side of length 6. The triangle also has a side of length 10. What is the length of the longest side of the triangle?

Sprint 7 _____ Mark's goal is to plant 900 trees. He pilots a drone that can plant trees at a rate of 5 trees per minute. However, once he is halfway done, another drone that also plants trees at a rate of 5 trees per minute starts to work. How many minutes elapses from the time Mark starts planting trees to the time his goal is reached?

Sprint 9 _____ Amelia is working out a subtraction problem where she subtracts a three-digit number from 1624. Among the two numbers in the subtraction problem, no digits are repeated. Amelia always forgets to carry and borrow when she subtracts, but fortunately she does not need to borrow at any point when doing this subtraction problem and gets the correct answer. What answer does Amelia get?

Sprint 10 _____ A number is selected uniformly at random from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. What is the probability that the number is odd or a multiple of 3? Express your answer as a common fraction.

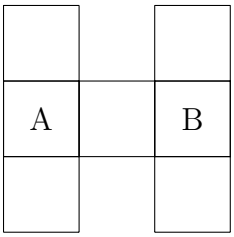
Sprint 11 _____ Brian is getting stronger! His power level is currently at 300, and every hour, his power level doubles. What is the smallest whole number of hours it would take for Brian's power level to go over 8000?

Sprint 12 (modified) _____ Thomas has a robot that can use a powerful gear technique, where each use lasts 5 seconds, but after each use, the robot needs to cool down before activating again. The cool down time for the first use is 10 seconds, but each subsequent cool down time is 10 seconds longer than that of the previous cool down time. How many times could the robot activate the gear technique within 2 minutes, assuming the robot first uses the gear technique as the timer starts?

Sprint 13 _____ At a party, there are 8 people. Each pair of people shakes hands with each other once, except for one pair, who are best friends and refuse to shake hands with anyone else other than themselves. In total, how many handshakes take place?

Sprint 20 _____ Audrey and her friends plan to visit Heaven's Park and Sakura Mountain, but they have to travel by vehicle, where each vehicle can have up to four passengers. The entrance fee for Heaven's Park is a fixed amount per vehicle, while the entrance fee for Sakura Mountain is a fixed amount per passenger. Audrey finds that the minimum cost if a group of 3 travels to both places is \$60 while the minimum cost if a group of 8 travels to both places is \$140. How many dollars does Audrey need if she, Melody, Lily, Jaedyn, and Madison all plan on travelling as a group to both places?

Sprint 27 (modified) _____ Jessica stands on square A and Rebecca stands on square B in the diagram below. At the same time, Jessica jumps to a square chosen at random among the squares that share a side with square A, and Rebecca jumps to a square chosen at random that shares a side with square B. What is the probability that they jump to the same square? Express your answer as a common fraction.



Sprint 30 (modified) _____ Shafiq is making a staircase out of blocks with at least one column where column 1 has 1 block, column 2 has 2 blocks, and so on. In general, column n should have n blocks. Shafiq could use up all his blocks to make a staircase if there were either 1 more block or 4 fewer blocks. What is the sum of all possible number of blocks in Shafiq's collection?

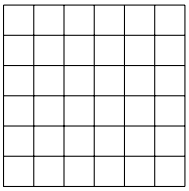
Target 1 _____ Mehuli is going on a road trip. She first travels 100 miles per day due north for seven days. Then she travels 30 miles per day due south for three days. At the end of the tenth day, how many miles due north is Mehuli from her starting point?

Target 2 _____ At the end of a soccer game, Claire's team and Cora's team line up in a line to

shake hands. Claire and Cora are standing 10 meters apart and facing each other. Both Claire and Cora have 14 teammates from their respective teams standing behind them such that the teammates of each team are standing 2 meters apart. How many meters is the farthest distance from one player on Claire's team to one player on Cora's team?

Target 5 _____ On a mathleague.org high school test, the Sprint portion is worth 120 points and the Target portion is worth 80 points. The lowest percentage score that one can earn on the Sprint test and still get an overall score of 70 percent is $n\%$. What is n?

Target 7 _____ The 6×6 rectangle below can be split into four rectangles that each have dimensions 1×5 as well as another rectangle with whole number side lengths. What is the perimeter of the fifth rectangle?



Team 1 _____ A lecture hall has 10 rows where each row has 10 seats, and each person who attends a lecture sits in a seat such that each seat has at most one student. Sofia keeps track of the number of students who attend lecture each day in the below table. In how many lectures do there remain an odd number of empty seats at that time?

Lecture Number	1st	2nd	3rd	4th	5th
Students Attending	96	79	68	53	41

Team 2 _____ Ethan pays for a ride with the transportation service Speedy, which comes at a base fee of \$15, plus \$1.50 for each mile traveled. He would have had to pay three times the amount if he had traveled four times as far as he did. How many miles did Ethan travel?

Team 6 _____ Kendrick has 15 acorns, 10 metal fragments, and 20 smooth stones available to make capsules. The below table lists the number of resources needed to make each capsule. What is the greatest number of capsules that Kendrick could make?

Capsule Type	Acorns Needed	Metal Fragments Needed	Smooth Stones Needed
Standard	1	1	1
Light	1	1	0
Heavy	1	0	2

Countdown 1 _____ Bella's bubble soccer team has 9 players, including Drake. She has to pick 5 players to be on the field, but she wants one of the players picked to be Drake. How many ways can Bella pick the rest of the players?

Countdown 3 _____ Matt is in a hall of mirrors that has 100 mirrors, each in a shape of a parallelogram. He observes that 56 mirrors are not rectangles, 42 mirrors are not rhombuses, and 14 mirrors are squares. How many mirrors are rectangles but not squares?

Countdown 13 _____ JR divides a rectangular field with area 240 square feet into smaller plots that are squares that each have area 16 square feet. The tomato plots do not share an edge with the edge of the entire rectangular field, and the total area of the tomato plots is 48 square feet. What is the perimeter of JR's rectangular field?

Countdown 16 _____ Sofia needs to obtain the books shown in the below table. Fortunately, a friend gave her used copies of the two most expensive books on the list, so Sofia only needs to buy the rest of the books. If Sofia has $3.28 left after buying the books, how many dollars does she have before buying the books? Express your answer as a decimal to the nearest hundredth.

Book	Cost
Blazing World	$12.99
Fahrenheit 451	$15.99
Frankenstein	$8.99
Secret Life of Bees	$11.99
The Odyssey	$10.99

Countdown 19 (modified) _____ A circle is divided into a number of pieces by 4 straight lines, where each pair of lines is either parallel or perpendicular. What is the maximum number of pieces the circle is divided into?

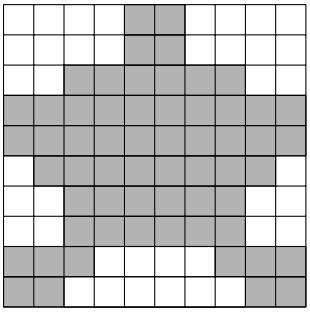
Countdown 20 _____ Eugene and Marian are working on a jigsaw puzzle, where the puzzle once completed has 20 rows with each row having 25 pieces. So far, Marian completed the space ranger part, which used up 20% of the total pieces. Then Eugene completes the tricky green slime monster part, which uses 37.5% of the pieces not used for the space ranger part. How many remaining pieces are left? (DMK: this question is not that interesting but shows how it can be useful to memorize the decimal/percent representations of eighths: $1/8 = 0.125 = 12.5\%$, $2/8 = 0.25 = 25\%$, $3/8 = 0.375 = 37.5\%$, etc.)

Countdown 21 (modified) _____ If $f(x) = 99x + 33$, then find $f(33)$. (DMK: note $99x$ means 99 times x.)

Countdown 22 (modified) _____ Consider the four whole numbers 98, 99, 100, and 101. Among these, what is the sum of all the numbers that are not divisible by 7?

Countdown 23 (modified) _____ A basketball game is 48 minutes long and has four quarters with each quarter lasting the same time. At the end of the third quarter, the Tunes have 73 points while the Monsters have 81 points. The Tunes average 3 points per minute in the fourth quarter, while the Monsters average 2 points per minute in the fourth quarter. In the end, who wins the game?

Countdown 30 (modified) _____ A large square is divided into 100 unit squares, as shown below. What percent of the area of the large square is shaded?



Countdown 35 (modified) _____ How many ways are there to draw a rectangle from a 4×2 grid of squares if the sides of the rectangle are on the lines of the grid?

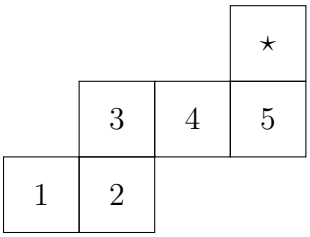
Countdown 37 _____ How many diagonals are there in a regular pentagon?

Countdown 38 (modified) _____ What is the least common multiple of the first four even positive integers? (DMK: I haven't included all of them, but there are many questions about "least common multiple" and/or "greatest common divisor.")

Countdown 42 (modified) _____ Tabitha went to sleep at 9:20pm and woke up at 6:30am the next day. She spent 20% of the time sleeping in deep sleep. How many minutes did Tabitha spend in deep sleep?

Countdown 46 _____ Two fair six-sided dice are thrown. What is the probability that at least one of them lands on a 2 or a 5? Express your answer as a common fraction.

Countdown 56 (modified) _____ When the figure shown below is folded into a cube such that the squares shown become faces of the cube, what is the sum of all the numbers on all the faces that share an edge with the face marked with a $\star$?



Countdown 63 (modified) _____ There are 5 points on a circle. How many ways are there to draw a triangle with vertices chosen from the 5 points?

Countdown 70 (modified) _____ 20% of what number is 50% of 24?

Countdown 75 (modified) _____ In how many ways can the letters in the word OREO be arranged?

Countdown 76 _____ What is the maximum number of acute angles that can be formed from four lines that intersect in a single point?

Countdown 78 (modified) _____ How many proper subsets does the set $\{1, 2, 4\}$ have?